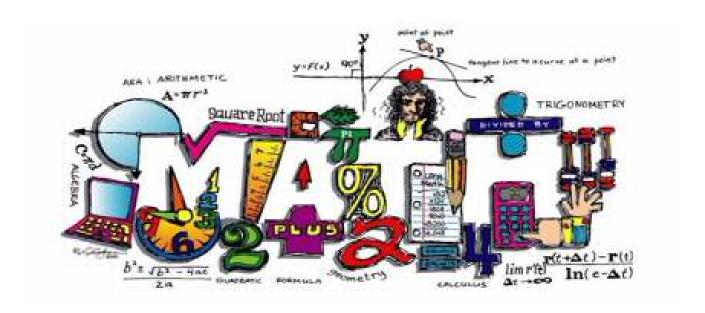
Name	Period

# **Algebra 1 Honors Incoming Assignment**

\*\*\*\*

**Bak MSOA Summer Required Mathematics Assignment Directions:** 



## **NO CALCULATOR!**

Complete the ODD Problems, only, on each page.

Show all appropriate work and circle your answers.

The packet collection will not occur during the first day of school.

This will be a part of your first nine weeks Assignment grade.

DATE \_\_\_\_\_ PERIOD \_\_\_\_

# Divide Rational Numbers

Two numbers with a product of 1 are multiplicative inverses, or reciprocals, of each other.

**Example 1** Write the multiplicative inverse of  $-2\frac{3}{4}$ .

$$-2\frac{3}{4} = -\frac{11}{4}$$
 Write  $-2\frac{3}{4}$  as an improper fraction. Since  $-\frac{11}{4}\left(-\frac{4}{11}\right) = 1$ , the multiplicative inverse of  $-2\frac{3}{4}$  is  $-\frac{4}{11}$ .

To divide by a fraction, multiply by its multiplicative inverse.

**Example 2** Find  $\frac{3}{8} \div \frac{6}{7}$ . Write in simplest form.

$$\frac{3}{8} \div \frac{6}{7} = \frac{3}{8} \cdot \frac{7}{6}$$
 Multiply by the multiplicative inverse of  $\frac{6}{7}$ , which is  $\frac{7}{6}$ .
$$= \frac{\frac{1}{8}}{8} \cdot \frac{7}{6}$$
 Divide 6 and 3 by their GCF, 3.
$$= \frac{7}{16}$$
 Multiply.

Write the multiplicative inverse of each number.

1. 
$$\frac{3}{5}$$

**2.** 
$$-\frac{8}{9}$$

3. 
$$\frac{1}{10}$$

4. 
$$-\frac{1}{6}$$

5. 
$$2\frac{3}{5}$$

6. 
$$-1\frac{2}{3}$$

7. 
$$-5\frac{2}{5}$$

8. 
$$7\frac{1}{4}$$

Divide. Write in simplest form.

**9.** 
$$\frac{1}{3} \div \frac{1}{6}$$

10. 
$$\frac{2}{5} \div \frac{4}{7}$$

11. 
$$-\frac{5}{6} \div \frac{3}{4}$$

12. 
$$1\frac{1}{5} \div 2\frac{1}{4}$$

**13.** 
$$3\frac{1}{7} \div \left(-3\frac{2}{3}\right)$$

14. 
$$-\frac{4}{9} \div 2$$

15. 
$$\frac{6}{11} \div (-4)$$

**16.** 
$$5 \div 2\frac{1}{3}$$

# Variables and Expressions

To evaluate an algebraic expression you replace each variable with its numerical value, then use the order of operations to simplify.

## **Example 1** Evaluate 5m - 3n if m = 6 and n = -5.

$$5m - 3n = 5(6) - 3(-5)$$
  
=  $30 - (-15)$   
=  $45$ 

Replace m with 6 and n with -5.

Use the order of operations. Subtract -15 from 30.

**Example 2** Evaluate 
$$\left(\frac{3+ab}{3}\right)$$
 if  $a=7$  and  $b=6$ .

$$\frac{3+ab}{3} = \frac{3+(7)(6)}{3} = \frac{45}{3} = 15$$

Replace a with 7 and b with 6.

The fraction bar is like a grouping symbol.

Divide.

## Example 3 Evaluate 6(x + y) - 4 if x = 8 and y = 3.

$$6(x + y) - 4 = 6(8 + 3) - 4$$
$$= 6(11) - 4$$
$$= 62$$

Replace x with 8 and y with 3.

Use the order of operations.

Subtract 4 from 66.

#### Example 4 Translate each phrase into an algebraic expression.

a. twelve dollars less than Tamika has

Let d represent the money Tamika has. The expression is d-12.

**b.** three less than twice the number of students

Let *n* represent the number of students. The expression is 2n-3.

#### **Exercises**

Evaluate each expression if a = 4, b = 2, and c = -3.

**4.** 
$$5a + 6c$$

**5.** 
$$\frac{ab}{8}$$

**6.** 
$$2a - 3b$$

**7.** 
$$\frac{ac}{b}$$

**9.** 
$$20 - \frac{ac}{b} + 2$$

**11.** 
$$\frac{ac - 3b}{b}$$

12. 
$$\frac{6+3b}{2a-2}$$

Translate each phrase into an algebraic expression.

13. four more than the number of DVDs Joan has

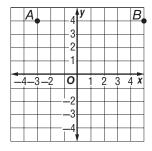
14. six less than five times the number of miles

## Ordered Pairs and Relations

**Example 1** Name the ordered pair for point A.

- Start at the origin.
- Move left on the *x*-axis to find the *x*-coordinate of point *A*, which is -3.
- Move up the *y*-axis to find the *y*-coordinate, which is 4.

So, the ordered pair for point A is (-3, 4).



**Example 2** Graph point B at (5, 4).

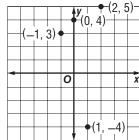
- Use the coordinate plane shown above. Start at the origin and move 5 units to the right. Then move up 4 units.
- Draw a dot and label it B(5, 4).

**Example 3** Express the relation  $\{(2, 5), (-1, 3), (0, 4), (1, -4)\}$  as a table and a graph. Then state the domain and range.

The domain is  $\{-1, 0, 1, 2\}$ .

The range is  $\{-4, 3, 4, 5\}$ .

$\boldsymbol{x}$	y
2	5
-1	3
0	4
1	-4



#### **Exercises**

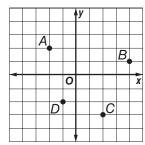
Name the ordered pair for each point.

**1.** A

**2.** *B* 

**3.** C

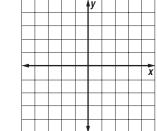
**4.** D



Express the relation as a table and a graph. Then state the domain and range.

**5.** 
$$\{(-3, 1), (2, 4), (-1, 0), (4, -4)\}$$

x	y



## **Functions**

A function is a relation in which each member of the domain (input value) is paired with exactly one member of the range (output value). You can organize the input, rule, and output of a function using a function table.

**Example 1** Choose four values for x to make a function table for f(x) = 2x + 4. Then state the domain and range of the function.

Substitute each domain value *x*, into the function rule. Then simplify to find the range value.

$$f(x) = 2x + 4$$

$$f(-1) = 2(-1) + 4$$
 or 2

$$f(0) = 2(0) + 4 \text{ or } 4$$

$$f(1) = 2(1) + 4$$
 or 6

$$f(2) = 2(2) + 4$$
 or 8

The domain is  $\{-1, 0, 1, 2\}$ . The range is  $\{2, 4, 6, 8\}$ .

Input x	Rule $2x + 4$	Output f(x)
-1	2(-1) + 4	2
0	2(0) + 4	4
1	2(1) + 4	6
2	2(2) + 4	8

Exercises

Find each function value.

**1.** 
$$f(1)$$
 if  $f(x) = x + 3$ 

**2.** 
$$f(6)$$
 if  $f(x) = 2x$ 

**2.** 
$$f(6)$$
 if  $f(x) = 2x$  **3.**  $f(4)$  if  $f(x) = 5x - 4$ 

**4.** 
$$f(9)$$
 if  $f(x) = -3x + 10$  **5.**  $f(-2)$  if  $f(x) = 4x - 1$ 

**5.** 
$$f(-2)$$
 if  $f(x) = 4x - 1$ 

**6.** 
$$f(-5)$$
 if  $f(x) = -2x + 8$ 

Choose four values for x to make a function table for each function. Then state the domain and range of the function.

7. 
$$f(x) = x - 10$$

**8.** 
$$f(x) = 2x + 6$$

**9.** 
$$f(x) = 2 - 3x$$

x	x - 10	f(x)

x	2x + 6	f(x)

$\boldsymbol{x}$	2-3x	J(x)

## **Linear Functions**

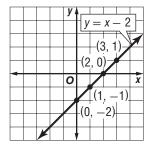
A function in which the graph of the solutions forms a line is called a linear function. A linear function can be represented by an equation, a table, a set of ordered pairs, or a graph.

Example 1 Graph y = x - 2.

**Step 1** Choose some values for x. Use these values to make a function table.

x	x-2	у	(x, y)
0	0 - 2	-2	(0, -2)
1	1 - 2	-1	(1, -1)
2	2 - 2	0	(2, 0)
3	3 - 2	1	(3, 1)

**Step 2** Graph each ordered pair on a coordinate plane. Draw a line that passes through the points. The line is the graph of the linear function.

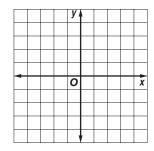


#### **Exercises**

Complete the function table. Then graph the function.

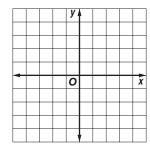
1. 
$$y = x + 3$$

$\boldsymbol{x}$	x + 3	у	(x, y)
-2			
0			
1			
2			

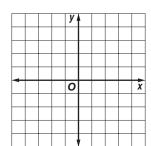


Graph each function.

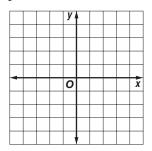
**2.** 
$$y = 3x + 2$$



3. 
$$y = 2 - x$$



**4.** 
$$y = 3x - 1$$



Determine whether each set of data is continuous or discrete.

- 5. the size of airmail packages
- **6.** the number of boxes in an airmail shipment

## Linear and Nonlinear Functions

Linear functions represent constant rates of change. The rate of change for nonlinear functions is not constant. That is, the values do not increase or decrease at the same rate. You can use a table to determine if the rate of change is constant.

#### Example 1

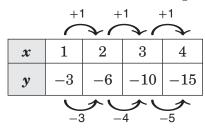
Determine whether the table represents a linear or a nonlinear function. Explain.

	+2	+2	2 -	+2
		<b>&gt;</b> (	71	<b>→</b>
x	3	5	7	9
y	7	10	13	16
+3 +3 +3				

As *x* increases by 2, *y* increases by 3. The rate of change is constant, so this function is linear.

#### Example 2

Determine whether the table represents a linear or a nonlinear function. Explain.



As *x* increases by 1, *y* decreases by a different amount each time. The rate of change is not constant, so this function is nonlinear.

#### **Exercises**

Determine whether each table represents a linear or a nonlinear function. Explain.

1.

•	x	3	5	7	9
	у	7	9	11	13

2.

x	1	5	9	13
y	0	6	8	9

3

•	x	3	6	9	12
	y	2	3	4	5

<b>1.</b>	x	-2	-3	-4	-5
	y	-1	-5	9	8

# **Constant Rate of Change**

Relationships that have straight-lined graphs are called linear relationships. The rate of change between any two points in a linear relationship is the same, or constant. A linear relationship has a constant rate of change.

Example

The height of a hot air balloon after a few seconds is shown. Determine whether the relationship between the two quantities is linear. If so, find the constant rate of change. If not, explain your reasoning.

	Time (sec)	Height of Hot Air Balloon (ft)	
+1 (	1	9	) +9
	2	18	<del>   </del>
+1	3	27	<b> </b> ← 9
+1 (	4	36	<b>)</b> +9

As the number of seconds increase by 1, the height of the balloon increases by 9 feet.

Since the rate of change is constant, this is a linear relationship. The constant rate of change is  $\frac{9}{1}$  or 9 feet per second. This means that the balloon is rising 9 feet per second.

#### Exercises

Determine whether the relationship between the two quantities described in each table is linear. If so, find the constant rate of change. If not, explain your reasoning.

1.

Greeting Cards				
Number of Cards   Total Cost(\$)				
1	1.50			
2	3.00			
3	4.50			
4	6.00			

2.

Party Table Rental		
Number of Tables	Cost(\$)	
1	10	
2	18	
3	24	
4	28	

•	Donuts			
	<b>Dozens Bought</b>	Cost (\$)		
	2	3.25		
	4	6.50		
	6	9.75		
	8	13.00		

•	Running				
	Time (min)	Distance(mi)			
	15	2			
	30	4			
	45	5			
	60	6			

# Slope

The slope m of a line passing through points  $(x_1, y_1)$  and  $(x_2, y_2)$  is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates. As an equation, the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
, where  $x_1 \neq x_2$ .

**Example 1** Find the slope of the line that passes through A(-1, -1) and B(2, 3).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

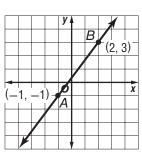
$$m = \frac{1}{x_2 - x_1}$$

$$m = \frac{3 - (-1)}{2 - (-1)}$$

 $(x_1, y_1) = (-1, -1),$   $(x_2, y_2) = (2, 3)$ 

$$m = \frac{4}{3}$$

Simplify.



**Check** When going from left to right, the graph of the line slants upward. This is correct for a positive slope.

#### Example 2

Find the slope of the line that passes through C(1,4) and D(3,-2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

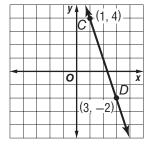
Slope formula

$$m = \frac{-2-4}{3-1}$$

 $(x_1, y_1) = (1, 4),$   $(x_2, y_2) = (3, -2)$ 

$$m = \frac{-6}{2}$$
 or  $-3$ 

Simplify.



**Check** When going from left to right, the graph of the line slants downward. This is correct for a negative slope.

#### Exercises

Find the slope of the line that passes through each pair of points.

**1.** 
$$A(0, 1), B(3, 4)$$

**2.** 
$$C(1, -2), D(3, 2)$$

3. 
$$E(4, -4), F(2, 2)$$

**6.** 
$$K(-4, 4), L(5, 4)$$

# Slope-Intercept Form

Linear equations are often written in the form y = mx + b. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

#### Example 1

State the slope and the y-intercept of the graph of y = x - 3.

$$y = x - 3$$
$$y = 1x + (-3)$$

Write the original equation.

$$y = 1x + (-3)$$

Write the equation in the form y = mx + b.

$$\uparrow \qquad \uparrow \\
v = mx + b$$

m = 1, b = -3

The slope of the graph is 1, and the *y*-intercept is -3.

You can use the slope-intercept form of an equation to graph the equation.

**Example 2** Graph y = 2x + 1 using the slope and y-intercept.

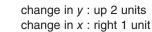
**Step 1** Find the slope and *y*-intercept.

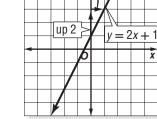
y = 2x + 1slope = 2, y-intercept = 1

**Step 2** Graph the *y*-intercept 1.

**Step 3** Write the slope 2 as  $\frac{2}{1}$ . Use it to locate a second point on the line.

$$m = \frac{2}{1} \leftarrow$$





right 1

**Step 4** Draw a line through the two points.

#### Exercises

State the slope and the y-intercept for the graph of each equation.

1. 
$$y = x + 1$$

**2.** 
$$y = 2x - 4$$

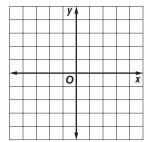
**3.** 
$$y = \frac{1}{2}x - 1$$

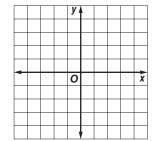
Graph each equation using the slope and the y-intercept.

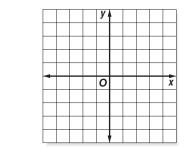
**4.** 
$$y = 2x + 2$$

**5.** 
$$y = x - 1$$

**6.** 
$$y = \frac{1}{2}x + 2$$



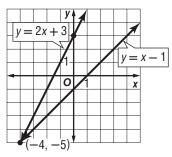




# Solve Systems of Equations by Graphing

**Example** Solve the system y = 2x + 3 and y = x - 1 by graphing.

Graph each equation on the same coordinate plane.



The graphs appear to intersect at (-4, -5).

Check this estimate by replacing x with -4 and y with -5.

Check

$$y = 2x + 3$$

$$y = x - 1$$

$$-5 \stackrel{?}{=} 2(-4) + 3$$

$$-5 \stackrel{?}{=} -4 - 1$$

$$-5 = -5$$
 ✓

$$-5 = -5 \checkmark$$

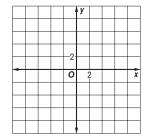
The solution of the system is (-4, -5).

#### Exercises

Solve each system of equations by graphing.

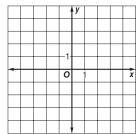
1. 
$$y = 2x + 5$$

$$y = -x + 8$$



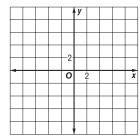
**2.** 
$$y = -x - 3$$

$$y = x + 1$$



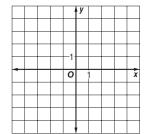
3. 
$$y = -3x + 9$$

$$y = -3x + 3$$



**4.** 
$$y = -2x + 4$$

$$y = -x + 3$$



# Write Equations

The table shows several verbal phrases for each algebraic expression.

Phrases	Expression	Phrases Expres		
8 more than a number the sum of 8 and a number <i>x</i> plus 8 <i>x</i> increased by 8	x + 8	the difference of $r$ and 6 6 subtracted from a number 6 less than a number $r$ minus 6	r-6	
Phrases	Expression	Phrases	Expression	
4 multiplied by <i>n</i> 4 times a number the product of 4 and <i>n</i>	4n	a number divided by 3 the quotient of <i>z</i> and 3 the ratio of <i>z</i> and 3	$\frac{z}{3}$	

The table shows several verbal sentences that represent the same equation.

Sentences	Equation
9 less than a number is equal to 45. The difference of a number and 9 is 45. A number decreased by 9 is 45. 45 is equal to a number minus 9.	n - 9 = 45

### Write each verbal phrase as an algebraic expression.

**1.** the sum of 8 and t

**2.** the quotient of g and 15

**3.** the product of 5 and b

**4.** *p* increased by 10

**5.** 14 less than *f* 

**6.** the difference of 32 and x

## Define a variable. Then write an equation to model each situation.

- 7. 5 more than a number is 6.
- **8.** The product of 7 and b is equal to 63.
- **9.** The sum of r and 45 is 79.
- **10.** The quotient of x and 7 is equal to 13.
- **11.** The original price decreased by \$5 is \$34.
- **12.** 5 shirts at \$*d* each is \$105.65.

# Solve Addition and Subtraction Equations

You can use the following properties to solve addition and subtraction equations.

- Addition Property of Equality If you add the same number to each side of an equation, the two sides remain equal.
- Subtraction Property of Equality If you subtract the same number from each side of an equation, the two sides remain equal.

**Example 1** Solve w + 19 = 45. Check your solution.

$$w + 19 = 45$$

Write the equation.

$$-19 = -19$$

Subtraction Property of Equality

$$w = 26$$

Simplify.

**Check** 
$$w + 19 = 45$$

Write the original equation.

$$26 + 19 \stackrel{?}{=} 45$$

Replace w with 26. Is this sentence true?

$$45 = 45 \checkmark$$

The sentence is true.

**Example 2** Solve h - 25 = -76. Check your solution.

$$h - 25 = -76$$

Write the equation.

$$+25 = +25$$

Addition Property of Equality

$$h = -51$$

Simplify.

**Check** 
$$h - 25 = -76$$

Write the original equation.

$$-51 - 25 \stackrel{?}{=} -76$$

Replace *h* with -51. Is this sentence true?

$$-76 = -76$$
  $\checkmark$ 

The sentence is true.

#### Exercises

Solve each equation. Check your solution.

1. 
$$s - 4 = 12$$

**2.** 
$$d + 2 = 21$$

**3.** 
$$h + 6 = 15$$

**4.** 
$$x + 5 = -8$$

**5.** 
$$b - 10 = -34$$

**6.** 
$$f - 22 = -6$$

7. 
$$17 + c = 41$$

8. 
$$v - 36 = 25$$

**9.** 
$$y - 29 = -51$$

**10.** 
$$19 = z - 32$$

11. 
$$13 + t = -29$$

**12.** 
$$55 = 39 + k$$

**13.** 
$$62 + b = 45$$

**14.** 
$$x - 39 = -65$$

**15.** 
$$-56 = -47 + n$$

# Solve Multiplication and Division Equations

You can use the following properties to solve multiplication and division equations.

- Multiplication Property of Equality If you multiply each side of an equation by the same number, the two sides remain equal.
- Division Property of Equality If you divide each side of an equation by the same nonzero number, the two sides remain equal.

**Example 1** Solve 19w = 114. Check your solution.

$$19w = 114$$

$$\frac{19w}{19} = \frac{114}{19}$$

Division Property of Equality

$$w = 6$$

Simplify.

Check

$$19w = 114$$

Write the original equation.

$$19(6) \stackrel{?}{=} 114$$

Replace w with 6.

$$114 = 114 \checkmark$$

This sentence is true.

Example 2 Solve  $\frac{d}{15} = -9$ . Check your solution.

$$\frac{d}{15} = -9$$

Write the equation.

$$\frac{d}{15}(15) = -9(15)$$

Multiplication Property of Equality

$$d = -135$$

Simplify.

Check

$$\frac{d}{15} = -9$$

Write the original equation.

$$\frac{-135}{15} \stackrel{?}{=} -9$$

Replace d with -135.

$$-9 = -9 \checkmark$$

This sentence is true.

#### Exercises

Solve each equation. Check your solution.

1. 
$$\frac{r}{5} = 6$$

**2.** 
$$2d = 12$$

3. 
$$7h = -21$$

**4.** 
$$-8x = 40$$

**5.** 
$$\frac{f}{8} = -6$$

6. 
$$\frac{x}{-10} = -7$$

7. 
$$17c = -68$$

8. 
$$\frac{h}{-11} = 12$$

**9.** 
$$29t = -145$$

**10.** 
$$125 = 5z$$

11. 
$$13t = -182$$

12. 
$$117 = -39k$$

# Solve Two-Step Equations

A two-step equation contains two operations. To solve a two-step equation, undo each operation in reverse order.

Example 1 Solve -2a + 6 = 14. Check your solution.

$$-2a + 6 = 14$$

Write the equation.

$$-6 = -6$$

Subtraction Property of Equality

$$-2a = 8$$

Simplify.

$$\frac{-2a}{-2} = \frac{8}{-2}$$

Division Property of Equality

$$a = -4$$

Simplify.

Check

$$-2a + 6 = 14$$

Write the equation.

$$-2(-4) + 6 \stackrel{?}{=} 14$$

Replace a with -4 to see if the sentence is true.

$$14 = 14 \checkmark$$
 The sentence is true.

The solution is -4.

Sometimes it is necessary to combine like terms before solving an equation.

Example 2 Solve 5 = 8x - 2x - 7. Check your solution.

$$5 = 8x - 2x - 7$$

Write the equation.

$$5 = 6x - 7$$

Combine like terms.

$$5 + 7 = 6x - 7 + 7$$

Addition Property of Equality

$$12 = 6x$$

Simplify.

$$\frac{12}{6} = \frac{6x}{6}$$

Division Property of Equality

Simplify.

The solution is 2.

Check this solution.

#### Exercises

Solve each equation. Check your solution.

1. 
$$2d + 7 = 9$$

**2.** 
$$11 = 3z + 5$$

3. 
$$2s - 4 = 6$$

4. 
$$-12 = 5r + 8$$

**5.** 
$$-6p - 3 = 9$$

**6.** 
$$-14 = 3x + x - 2$$

7. 
$$5c + 2 - 3c = 10$$

8. 
$$3 + 7n + 2n = 21$$

**9.** 
$$21 = 6r + 5 - 7r$$

**10.** 
$$8 - 5b = -7$$

11. 
$$-10 = 6 - 4m$$

12. 
$$-3t + 4 = 19$$

**13.** 
$$2 + \frac{a}{6} = 5$$

**14.** 
$$-\frac{1}{3}q - 7 = -3$$

**15.** 
$$4 - \frac{v}{5} = 0$$

# Write Two-Step Equations

Some verbal sentences translate into two-step equations.

### Example 1

Translate each sentence into an equation.

Sentence	Equation

Four more than three times a number is 19. 
$$3n + 4 = 19$$

Five is seven less than twice a number. 
$$5 = 2n - 7$$

Seven more than the quotient of a number and 3 is 10. 
$$7 + \frac{n}{3} = 10$$

After a sentence has been translated into a two-step equation, you can solve the equation.

### Example 2

Translate the sentence into an equation. Then find the number. Thirteen more than five times a number is 28.

Words Thirteen more than five times a number is 28.

Variable Let n = the number.

$$5n + 13 = 28$$

$$-13 = -13$$

$$5n = 15$$
$$\frac{5n}{5} = \frac{15}{3}$$

$$\frac{3n}{5} = \frac{1}{3}$$

Therefore, the number is 3.

#### Exercises

Translate each sentence into an equation. Then find each number.

- **1.** Five more than twice a number is 7.
- **2.** Fourteen more than three times a number is 2.
- **3.** Seven less than twice a number is 5.
- **4.** Two more than four times a number is -10.
- **5.** Eight less than three times a number is -14.
- **6.** Three more than the quotient of a number and 2 is 7.

# **Powers and Exponents**

The product of repeated factors can be expressed as a power. A power consists of a base and an **exponent**. The exponent tells how many times the base is used as a factor.

**Example 1** Write each expression using exponents.

$$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$$

The number 7 is a factor 4 times. So, 7 is the base and 4 is the exponent.

b. 
$$y \cdot y \cdot x \cdot y \cdot x$$

$$y \cdot y \cdot x \cdot y \cdot x = y \cdot y \cdot y \cdot x \cdot x$$

Commutative Property

$$= (y \cdot y \cdot y) \cdot (x \cdot x)$$

Associative Property

$$= y^3 \cdot x^2$$

Definition of exponents

To evaluate a power, perform the repeated multiplication to find the product.

Example 2 Evaluate  $(-6)^4$ .

$$(-6)^4 = (-6) \cdot (-6) \cdot (-6) \cdot (-6)$$

Write the power as a product.

$$= 1,296$$

Multiply.

The order of operations states that exponents are evaluated before multiplication, division, addition, and subtraction.

Example 3 Evaluate  $m^2 + (n - m)^3$  if m = -3 and n = 2.

$$m^2 + (n - m)^3 = (-3)^2 + (2 - (-3))^3$$

Replace m with -3 and n with 2.

$$=(-3)^2+(5)^3$$

Perform operations inside parentheses.

$$= (-3\boldsymbol{\cdot} -3) + (5\boldsymbol{\cdot} 5\boldsymbol{\cdot} 5)$$

Write the powers as products.

$$= 9 + 125 \text{ or } 134$$

Add.

Exercises

Write each expression using exponents.

2. 
$$a \cdot a \cdot a \cdot a \cdot a \cdot a$$

Evaluate each expression.

**5.** 
$$(-3)^5$$

**6.** 
$$\left(\frac{3}{4}\right)^3$$

ALGEBRA Evaluate each expression if a = 5 and b = -4.

7. 
$$a^2 + b^2$$

**8.** 
$$(a + b)^2$$

**9.** 
$$a + b^2$$

# **Multiply and Divide Monomials**

The Product of Powers rule states that to multiply powers with the same base, add their exponents.

**Example 1** Simplify. Express using exponents.

a. 
$$2^3 \cdot 2^2$$

$$2^3 \cdot 2^2 = 2^{3+2}$$

The common base is 2.

$$= 2^{5}$$

Add the exponents.

b. 
$$2s^6(7s^7)$$

$$2s^{6}(7s^{7})=(2\,\boldsymbol{\cdot}\,7)(s^{6}\,\boldsymbol{\cdot}\,s^{7})$$

Commutative and Associative Properties

$$= 14(s^{6+7})$$

The common base is s.

$$= 14s^{13}$$

Add the exponents.

The Quotient of Powers rule states that to divide powers with the same base, subtract their exponents.

**Example 2** Simplify  $\frac{k^8}{k}$ . Express using exponents.

$$rac{k^8}{k^1} = k^{8-1}$$
 The common base is  $k$ .

Subtract the exponents.

Example 3 Simplify  $\frac{(-2)^{10} \cdot 5^6 \cdot 6^3}{(-2)^6 \cdot 5^3 \cdot 6^2}$ .

$$\frac{(-2)^{10} \cdot 5^6 \cdot 6^3}{(-2)^6 \cdot 5^3 \cdot 6^2} = \left(\frac{(-2)^{10}}{(-2)^6}\right) \cdot \left(\frac{5^6}{5^3}\right) \cdot \left(\frac{6^3}{6^2}\right)$$

Group by common base.

$$= (-2)^4 \cdot 5^3 \cdot 6^1$$

Subtract the exponents.

$$= 16 \cdot 125 \cdot 6 \text{ or } 12,000$$

Simplify.

#### Exercises

Simplify. Express using exponents.

1. 
$$5^2 \cdot 5^5$$

**2.** 
$$e^2 \cdot e^7$$

3. 
$$2a^5 \cdot 6a$$

**4.** 
$$4x^2(-5x^6)$$

5. 
$$\frac{7^9}{7^3}$$

**6.** 
$$\frac{v^{14}}{v^6}$$

7. 
$$\frac{15w^7}{5w^2}$$

8. 
$$\frac{10m^8}{2m}$$

9. 
$$\frac{2^5 \cdot 3^7 \cdot 4^3}{2^1 \cdot 3^5 \cdot 4}$$

10. 
$$\frac{4^{15} \cdot (-5)^6}{4^{12} \cdot (-5)^4}$$

## Powers of Monomials

Power of a Power: To find the power of a power, multiply the exponents.

Power of a Product: To find the power of a product, find the power of each factor and multiply.

Example 1

Simplify  $(5^3)^6$ .

 $(5^3)^6 = 5^3 \cdot 6$ Power of a power

 $=5^{18}$ Simplify.

**Example 2** Simplify  $(-3m^2n^4)^3$ .

 $(-3m^2n^4)^3 = (-3)^3 \cdot m^{2 \cdot 3} \cdot n^{4 \cdot 3}$ 

Power of a product

 $=-27m^6n^{12}$ 

Simplify.

### Exercises

Simplify.

1.  $(4^3)^5$ 

**2.**  $(4^2)^7$ 

3.  $(9^2)^4$ 

**4.**  $(k^4)^2$ 

**5.**  $[(6^3)^2]^2$ 

**6.**  $[(3^2)^2]^3$ 

7.  $(5q^4r^2)^5$ 

**8.**  $(3y^2z^2)^6$ 

**9.**  $(7a^4b^3c^7)^2$ 

**10.**  $(-4d^3e^5)^2$ 

**11.**  $(-5g^4h^9)^7$ 

**12.**  $(0.2k^8)^2$ 

# **Negative Exponents**

Any nonzero number to the zero power is 1. Any nonzero number to the negative n power is the multiplicative inverse of the number to the nth power.

## **Example 1** Write each expression using a positive exponent.

$$7^{-3} = \frac{1}{7^3}$$
 Definition of negative exponent

**b.**  $a^{-4}$ 

$$a^{-4} = \frac{1}{a^4}$$

Definition of negative exponent

## **Example 2** Evaluate each expression.

$$5^{-4}=rac{1}{5^4}$$
 Definition of negative exponent 
$$=rac{1}{625} \ 5^4=5 \cdot 5 \cdot 5 \cdot 5$$

**b.** 
$$(-3)^{-5}$$

$$(-3)^{-5} = \frac{1}{(-3)^5}$$
 Definition of negative exponent 
$$= \frac{1}{-243} \qquad (-3)^5 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)$$

#### Example 3

# Write $\frac{1}{\kappa^5}$ as an expression using a negative exponent.

$$\frac{1}{6^5} = 6^{-5}$$

Definition of negative exponent

## **Example 4** Simplify. Express using positive exponents.

a. 
$$x^{-3} \cdot x^5$$

$$x^{-3}$$
•  $x^5 = x^{(-3)+5}$  Product of Powers  $= x^2$  Add the exponents.

b. 
$$\frac{w^{-5}}{w^{-7}}$$

$$\frac{w^{-5}}{w^{-7}} = w^{-5 - (-7)}$$
 Quotient of Powers  $= w^2$  Subtract the expone

Subtract the exponents.

### Exercises

## Write each expression using a positive exponent.

1. 
$$a^{-8}$$

3. 
$$n^{-4}$$

## Evaluate each expression.

6. 
$$(-2)^{-5}$$

## Write each fraction as an expression using a negative exponent.

7. 
$$\frac{1}{5^7}$$

8. 
$$\frac{1}{3^6}$$

**9.** 
$$\frac{1}{x^8}$$

## Simplify. Express using positive exponents.

11. 
$$r^{-3} \cdot r^5$$

12. 
$$\frac{h^{-2}}{h^4}$$

# **Compare Real Numbers**

Numbers may be classified by identifying to which of the following sets they belong.

**Whole Numbers** 

**Rational Numbers** 

numbers that can be expressed in the form  $\frac{a}{b}$ , where a and b are

integers and  $b \neq 0$ 

**Irrational Numbers** 

numbers that cannot be expressed in the form  $\frac{a}{b}$ , where a and b

are integers and  $b \neq 0$ 

#### **Examples**

Name all sets of numbers to which each real number belongs.



whole number, integer, rational number

0.666...

Decimals that terminate or repeat are rational numbers, since they can be expressed as fractions.

Since  $-\sqrt{25} = -5$ , it is an integer and a rational number.

 $\sqrt{11} \approx 3.31662479...$  Since the decimal does not terminate or repeat, it is an irrational number.

To compare real numbers, write each number as a decimal and then compare the decimal values.

 $\sqrt{11}$ 

**Example 5** Replace with <, >, or = to make  $2\frac{1}{4} \circ \sqrt{5}$  a true statement.

Write each number as a decimal.

$$2\frac{1}{4} = 2.25$$

$$\sqrt{5} \approx 2.236067...$$

Since 2.25 is greater than 2.236067...,  $2\frac{1}{4} > \sqrt{5}$ .

#### **Exercises**

Name all sets of numbers to which each real number belongs.

3. 
$$5\frac{4}{7}$$

**4.** 
$$\sqrt{21}$$

**6.** 
$$-\sqrt{9}$$

7. 
$$\frac{6}{3}$$

8. 
$$-\sqrt{101}$$

Replace each  $\bigcirc$  with <, >, or = to make a true statement.

**9.** 
$$2.7 \bigcirc \sqrt{7}$$

**10.** 
$$\sqrt{11}$$
 •  $3\frac{1}{2}$ 

**10.** 
$$\sqrt{11}$$
 •  $3\frac{1}{2}$  **11.**  $4\frac{1}{6}$  •  $\sqrt{17}$ 

**12.** 
$$3.\overline{8} \bigcirc \sqrt{15}$$

For use with the lesson "Apply Order of Operations"

#### GOAL Use the order of operations to evaluate expressions.

## Vocabulary

The **order of operations** was established to evaluate an expression involving more than one operation.

#### **Order of Operations**

- **STEP 1** Evaluate expressions inside grouping symbols.
- **STEP 2** Evaluate powers.
- **STEP 3 Multiply** and **divide** from left to right.
- STEP 4 Add and subtract from left to right.

## **EXAMPLE 1** Evaluate expressions

### Evaluate the expression $4^2 \cdot 5 - 6^2$ .

#### Solution

$$4^{2} \cdot 5 - 6^{2} = 16 \cdot 5 - 36$$
 Evaluate powers.  
=  $80 - 36$  Multiply.

## **Exercises for Example 1**

#### **Evaluate the expression.**

1. 
$$20 - 3^2 + 7$$

**2.** 
$$5 \cdot 2^3 \div 6$$

**3.** 
$$4 \cdot 6 - 21 \div 3$$

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### **EXAMPLE 2**

## **Evaluate expressions with grouping symbols**

## **Evaluate the expression.**

**a.** 
$$47 - 2(9 + 12)$$

**b.** 
$$6[2^3 + (13 - 8)]$$

#### **Solution**

**a.** 
$$47 - 2(9 + 12) = 47 - 2(21)$$

$$= 47 - 42$$

= 6[8 + 5]

= 6[13]

= 78

**b.**  $6[2^3 + (13 - 8)] = 6[8 + (13 - 8)]$ 

1.2

# Study Guide continued

For use with the lesson "Apply Order of Operations"

## **Exercises for Example 2**

**Evaluate the expression.** 

**4.** 
$$3(14-5)$$

**6.** 
$$(7+5)-(8+4)$$

**8.** 
$$4^2(2+8)$$

**5.** 
$$6(9-1^4)$$

7. 
$$(3^3-6) \div 3$$

**9.** 
$$9[15 \div (2+3)]$$

## **EXAMPLE 3** Evaluate an algebraic expression

Evaluate the expression  $\frac{4y+8}{2+y}$  when y=3.

**Solution** 

$$\frac{4y+8}{2+y} = \frac{4(3)+8}{2+3}$$
 Substitute 3 for y.  

$$= \frac{12+8}{2+3}$$
 Multiply.  

$$= \frac{20}{5}$$
 Add.  

$$= 4$$
 Divide.

## **Exercises for Example 3**

Evaluate the expression when w = 9.

**10.** 
$$17 + 3w$$

**11.** 
$$w^2 - 13$$

**12.** 
$$\frac{5w}{w+6}$$

**13.** 
$$7(13 - w)$$

**14.** 
$$2w^2 - 15$$

**15.** 
$$5w - \frac{1}{3}w$$



### **GOAL** Translate verbal phrases into expressions.

## **Vocabulary**

A **verbal model** describes a situation using words as labels and using math symbols to relate the words.

A **rate** is a fraction that compares two quantities measured in different units.

A unit rate is a rate whose fraction has a denominator of 1.

## **EXAMPLE 1** Translate verbal phrases into expressions

#### Translate the phrase into an expression.

- **a.** 8 more than the product of 5 times a number w
- **b.** The quotient of 11 and the sum of 7 and a number x
- **c.** The square of a number y decreased by 13

#### Solution

Verbal Phrase Expression a. 8 more than the product of 5 times a number 
$$w = 8 + 5w$$

b. The quotient of 11 and the sum of 7 and a number  $x = \frac{11}{7+x}$ 

c. The square of a number  $y$  decreased by 13  $y^2 - 13$ 

## **Exercises for Example 1**

### Translate the phrase into an expression.

- **1.** The difference of 3 times a number m and 5
- **2.** 26 divided by a number n
- **3.**  $\frac{1}{3}$  of a number p
- **4.** The sum of 9 and the square of a number k

LESSON 1.3 Study Guide continued For use with the lesson "Write Expressions"

## **EXAMPLE 2** Use a verbal model to write an expression

A student reads p pages of a 230-page book. Write an expression for the number of unread pages in the book.

**Solution** 

**STEP 2** Translate the verbal model

An expression that represents the number of unread pages in the book is 230 - p.

## **Exercises for Example 2**

Write an expression for the situation.

- **5.** Total cost of *n* notebooks if each notebook costs \$1.25
- **6.** The time it takes to get to school and home again if you walk 5 minutes to the bus stop and ride the bus for *m* minutes

### **EXAMPLE 3** Find a unit rate

An airport checks in 460 passengers in 5 hours. Find the unit rate.

Solution

$$\frac{460 \text{ passengers}}{5 \text{ hours}} = \frac{460 \text{ passengers} \div 5}{5 \text{ hours} \div 5} = \frac{92 \text{ passengers}}{1 \text{ hour}}$$

The unit rate is 92 passengers per hour.

## **Exercises for Example 3**

Find the unit rate.

7. 
$$\frac{129 \text{ miles}}{6 \text{ gallons}}$$

9. 
$$\frac{$28}{4 \text{ tickets}}$$

8. 
$$\frac{18 \text{ people}}{3 \text{ tables}}$$

10. 
$$\frac{1500 \text{ meters}}{7.5 \text{ minutes}}$$

# **Study Guide**

For use with the lesson "Write Equations and Inequalities"

# GOAL Translate verbal sentences into equations or inequalities.

# Vocabulary An open sentence is a mathematical statement that contains two

An **open sentence** is a mathematical statement that contains two expressions and a symbol that compares them.

An **equation** is an open sentence that contains the symbol =.

An **inequality** is an open sentence that contains one of the symbols  $\leq$ ,  $\leq$ , >, or  $\geq$ .

When you substitute a number for the variable in an open sentence, the resulting statement is either true or false. If the statement is true, the number is a **solution of the equation**, or a **solution of the inequality.** 

#### **EXAMPLE 1**

## Write equations and inequalities

#### Write an equation or an inequality.

- **a.** 8 times the quantity of 11 plus a number x is 112.
- **b.** The product of 7 and a number y is no more than 31.
- **c.** A number z is more than 8 and at most 15.

#### Solution

#### Verbal phrase

## **a.** 8 times the quantity of 11 plus a number x is 112.

**b.** The product of 7 and a number 
$$y$$
 is no more than 31.

## **Exercises for Example 1**

#### Write an equation or an inequality.

- **1.** The difference of 73 and a number x is 17.
- **2.** The product of 8 and the quantity of a number y plus 6 is less than 21.
- **3.** The quotient of a number w and 5 is at most 4.
- **4.** The sum of a number z and 2 is greater than 15 and less than 23.

Equation or inequality

8(11 + x) = 112

7*y*≤ 31

 $8 < z \le 15$ 

For use with the lesson "Write Equations and Inequalities"

#### **EXAMPLE 2**

## **Check possible solutions**

Check whether 5 is a solution of the equation or inequality.

<b>Equation/inequality</b>	
----------------------------	--

**Substitute** 
$$3(5) - 7 \stackrel{?}{=} 12$$

**a.** 
$$3x - 7 = 12$$

$$3(5) - 7 \stackrel{?}{=} 12$$

$$8 \neq 12 X$$

**b.** 
$$9 + 2x \le 23$$

$$9 + 2(5) \stackrel{?}{\leq} 23$$

5 is a solution.

## **Exercises for Example 2**

Check whether the given number is a solution of the equation or inequality.

**5.** 
$$13 + a = 17; 4$$

**6.** 
$$7b - 3 = 10; 2$$

**7.** 
$$4c < 15$$
; 3

**8.** 
$$21 - 3d \ge 11; 2$$

**9.** 
$$4g + 6 \le 14; 3$$

**10.** 
$$7 < m + 8 < 15$$
; 6

#### **EXAMPLE 3**

## Solve a multi-step problem

A soccer team is selling pizzas for \$6 each. Each pizza costs \$4 to make. The team has 10 players and wants to raise \$900 for equipment and uniforms. How many pizzas does the team need to sell? How many pizzas will each player sell if every player sells the same number of pizzas?

#### **Solution**

**STEP 1** Write a verbal model. Let *p* be the number of pizzas sold. Write an equation.

$$\begin{pmatrix} \text{Price of} & -\text{ Cost to make} \\ \text{pizza} & -\text{ each pizza} \end{pmatrix} \times \begin{pmatrix} \text{Number of} \\ \text{pizzas sold} \end{pmatrix} = \text{Profit}$$

$$p = 900$$

**STEP 2** Use mental math to solve the equation (6-4)p = 900, or 2p = 900. Think: 2 times what number is 900? Because 2(450) = 900, the solution is 450.

The team needs to sell 450 pizzas.

**STEP 3** Find the number of pizzas each player sells:  $\frac{450 \text{ pizzas}}{10 \text{ players}} = 45 \text{ pizzas per player}$ 

Each player will sell 45 pizzas.

## **Exercise for Example 3**

11. Your family is driving 188 miles to visit a relative. Your father drives 63 miles then stops for a break. How many more miles are left in the trip? Your father drives 50 miles per hour. How long will the remainder of the trip take? Write a verbal model for the situation, then solve.

For use with the lesson "Use a Problem Solving Plan"

#### GOAL

Use a problem solving plan to solve problems.

#### Vocabulary

A **formula** is an equation that relates two or more quantities.

#### A Problem-Solving Plan

**STEP 1 Read and Understand** Read the problem carefully. Identify what you know and what you want to find out.

**STEP 2** Make a Plan Decide on an approach to solving the problem.

**STEP 3** Solve the Problem Carry out your plan. Try a new approach if the first one isn't successful.

**STEP 4** Look Back Once you obtain an answer, check that it is reasonable.

#### **EXAMPLE 1**

#### Read a problem and make a plan

A group of people go to a play. Adult tickets cost \$8 and tickets for children under twelve years of age cost \$5. There are 4 children under twelve. The group spends \$44 for all the tickets. How many adults attended the play?

#### Solution

#### STEP 1 Read and Understand

What do you know?

You know the cost of each ticket, the number of children attending, and the total cost of the tickets.

What do you want to find out?

You want to find the number of adult tickets purchased.

#### STEP 2 Make a Plan

Use what you know to write a verbal model that represents what you want to find out. Then write an equation and solve it.

## **Exercise for Example 1**

# Identify what you know and what you need to find out. Do *not* solve the problem.

**1.** A salesman is reimbursed \$50 a day for food and lodging. He also receives \$.35 for each mile driven. He drives 124 miles and is reimbursed \$193.40. How many days was the trip?

## **EXAMPLE 2** Solve a problem and look back

Solve the problem in Example 1 by carrying out the plan. Then check your answer.

#### **Solution**

**STEP 3** Solve the Problem Write a verbal model. Then write an equation. Let *a* be the number of adult tickets purchased.

Cost of adult tickets • Number of adult tickets • Cost of children's ticket • Number of children's ticket = Total cost 
$$a + 5$$
 •  $a + 4$  = 44

The equation is 8a + 20 = 44. One way to solve the equation is to use the strategy *guess*, *check* and *revise*.

**Guess** a number that seems reasonable considering the total cost of \$44. Try 2.

$$8a + 20 = 44$$
 Write equation.  
 $8(2) + 20 \stackrel{?}{=} 44$  Substitute 2 for a.  
 $36 \neq 44 \times$  Simplify; 2 does not check.

Because 36 < 44, try a larger number. Try 3.

$$8a + 20 = 44$$
 Write equation.  
 $8(3) + 20 \stackrel{?}{=} 44$  Substitute 3 for a.  
 $44 = 44 \checkmark$  Simplify.

The group bought 3 adult tickets.

**STEP 4** Look Back Each adult ticket purchase adds \$8 to the total ticket cost. Make a table.

Number of adults	0	1	2	3	4
Total cost	\$20	\$28	\$36	\$44	\$52

The total cost is \$44 when 3 adult tickets are purchased. The answer in Step 3 is correct.

## **Exercise for Example 2**

Use a problem solving plan to solve the problem.

**2.** You have saved \$165 to buy a video camera that costs \$300. You plan to save \$15 each week. How many weeks will it take to save for the video camera?

Chapter Resource Book

# The Distributive Property

- To multiply a sum by a number, multiply each addend by the number outside the parentheses.
- a(b + c) = ab + ac
- (b + c)a = ba + ca

## **Example 1** Find $6 \times 38$ mentally using the Distributive Property.

$$6 \times 38 = 6(30 + 8)$$

Write 38 as 
$$30 + 8$$
.

$$=6(30)+6(8)$$

Distributive Property

$$= 180 + 48$$

Multiply mentally.

$$= 228$$

Add.

So, 
$$6 \times 38 = 228$$
.

## **Example 2** Use the Distributive Property to rewrite 4(x + 3).

$$4(x+3) = 4(x) + 4(3)$$

Distributive Property

$$= 4x + 12$$

Multiply.

So, 4(x + 3) can be rewritten as 4x + 12.

#### Exercises

## Find each product mentally. Show the steps you used.

**4.** 
$$8 \times 5.7$$

### Use the Distributive Property to rewrite each algebraic expression.

**5.** 
$$5(y + 4)$$

**6.** 
$$(7 + r)3$$

7. 
$$12(x+5)$$

**8.** 
$$(b + 2)9$$

**9.** 
$$4(4 + a)$$

**10.** 
$$9(7 + v)$$